САМАРСКИЙ УНИВЕРСИТЕТ
SAMARA UNIVERSITY

Practice session

Measurement of inertial characteristics of nanosatellites

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INTERNATIONAL SUMMER SPACE SCHOOL
FUTURE SPACE TECHNOLOGIES AND EXPERIMENTS IN SPACE

The approaches for determination the center of mass coordinates and inertia tensor of nanosatellite

- numerical calculation method based on Computer Aided Design (CAD) models
- experimental approach
  - angular acceleration methods
  - oscillation methods
    - compound pendulums
    - torsional
    - multifilar
- parameter identification methods
  - vibration tests
  - based on motion equations of an unconstrained object

The platform provides the estimation of the center of mass coordinates of a CubeSat 1U-3U nanosatellite weighing from 1 to 10 kg with an error of no more than 0.5 mm in the building coordinate system and the estimation of moments of inertia with an error of no more than 1.5%.

Belokonov I. V., Barinova E. V., Ivliev A. V., Kliuchnik V. N .and Timbai I. A. Device for determining the position of the center of mass and moments of inertia of objects patent # 2698536, Russia

The coordinate systems

(1) Torsional platform coordinate system (*SXYZ*)

The torsional platform coordinate system is established with its origin *S* located on the plate of the torsional platform. *SZ*-axis is aligned with the axis of rotation, up warded. *SX* and *SY* axes are in plane of the rotating plate (or faceplate) of the torsional platform.

(2) Reference coordinate system (*Oxyz*)

The second coordinate system, referred to Reference coordinate system, is used to describe the *CM* of nanosatellite. The coordinate system is fixed to the building coordinate system, which is aligned with the ribs of CubeSat. It's origin *O* is choosing at the any convenient point of nanosatellite, for example, one of the vertices. Let be *Ox*-axis is along the longest rib of the nanosatellite.

(3) Centroid coordinate system (*Cxyz*)

Moving the origin of the Reference coordinate system to the *CM* of the nanosatellite, the Centroid coordinate system is obtained. The direction of each axis (*Cxyz*) in Centroid coordinate system is the same as that in Reference coordinate system *Oxyz*.

The inertia tensor of the nanosatellite in Centroid coordinate system

$$I = \begin{pmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{xy} & I_y & -I_{yz} \\ -I_{xz} & -I_{yz} & I_z \end{pmatrix}. \quad (1)$$

Assuming that restoring torque of the spring is linearly proportional the angular displacement (φ) by a spring constant (k_S) and neglecting friction torque, for an object oscillating about a vertical axis aligned with the axis of rotation (*SZ*) and the center of mass of the body the equation of motion can be written in form

$$I_{SZ}\ddot{\varphi} + k_S\varphi = 0. \qquad (2)$$

where I_{SZ} is the moment of inertia of the system about *SZ*-axis, φ is the angular displacement, k_S is the spring constant. Eq. (2) is a second-order linear ODE and for initial displacement φ_0 and zero initial velocity ($\dot{\varphi}_0 = 0$) has solution

$$\varphi(t) = \varphi_0 \, cos\left(\sqrt{\frac{k_S}{I_{SZ}}}t\right). \quad (3)$$

The system has a natural frequency ω_n and a period of oscillation *T*

$$\omega_n = \sqrt{\frac{k_S}{I_{SZ}}}, \quad (4) \qquad T = 2\pi\sqrt{\frac{I_{SZ}}{k_S}}. \quad (5)$$

Using (5) we can determine the moment of inertia about z-axis for known spring constant k_S and period of oscillation T as

$$I_{SZ} = \frac{T^2 \cdot k_S}{4\pi^2}. \qquad (6)$$

Formula (6) allows us calculate the moment of inertia of the whole system about the axis aligned with the axis of rotation. If the center of mass of the nanosatellite does not align with the SZ-axis we can write

$$I_{SZ} = I_0 + I_z + m(X_C^2 + Y_C^2), \qquad (7)$$

where I_0 is the own moment of inertia of the torsional platform, X_C, Y_C are coordinates of the nanosatellite CM in the torsional platform coordinate system, m is the mass of nanosatellite.

The I_0 own moment of inertia of the torsional platform can be determined by

$$I_0 = \frac{{T_0}^2 \cdot k_S}{4\pi^2}, \qquad (8)$$

where T_0 is the period of the oscillation of the empty torsional platform.

The formula for estimation of the inertia moment of nanosatellite

$$I_z = \frac{\left(T^2 - {T_0}^2\right) \cdot k_S}{4\pi^2} - m(X_C^2 + Y_C^2). \qquad (9)$$

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In order to estimate the two coordinates of the center of mass we need to provide 3 measurements
(1) the nanosatellite initial position,
(2) the position, when nanosatellite is shifted to the distance ΔX along *SX*-axis only,
(3) the position, when nanosatellite is shifted to the distance ΔY along *SY*-axis only.
Thus, we get three periods of oscillation $T_1,\ T_2,\ T_3$

$$X_C = \frac{(T_2^2 - T_1^2)k_S}{8\pi^2 m\Delta X} - \frac{\Delta X}{2}, \qquad (10)$$

$$Y_C = \frac{(T_3^2 - T_1^2)k_S}{8\pi^2 m\Delta Y} - \frac{\Delta Y}{2}. \qquad (11)$$

If the axes of the Reference coordinate system and the torsional platform coordinate system are in the same direction, we can estimate the values $x_C,\ y_C$ by

$$x_C = X_C - X_O, \qquad (12)$$

$$y_C = Y_C - Y_O, \qquad (13)$$

where $X_O,\ Y_O$ are origin *O* coordinates in the Torsional platform coordinate system.

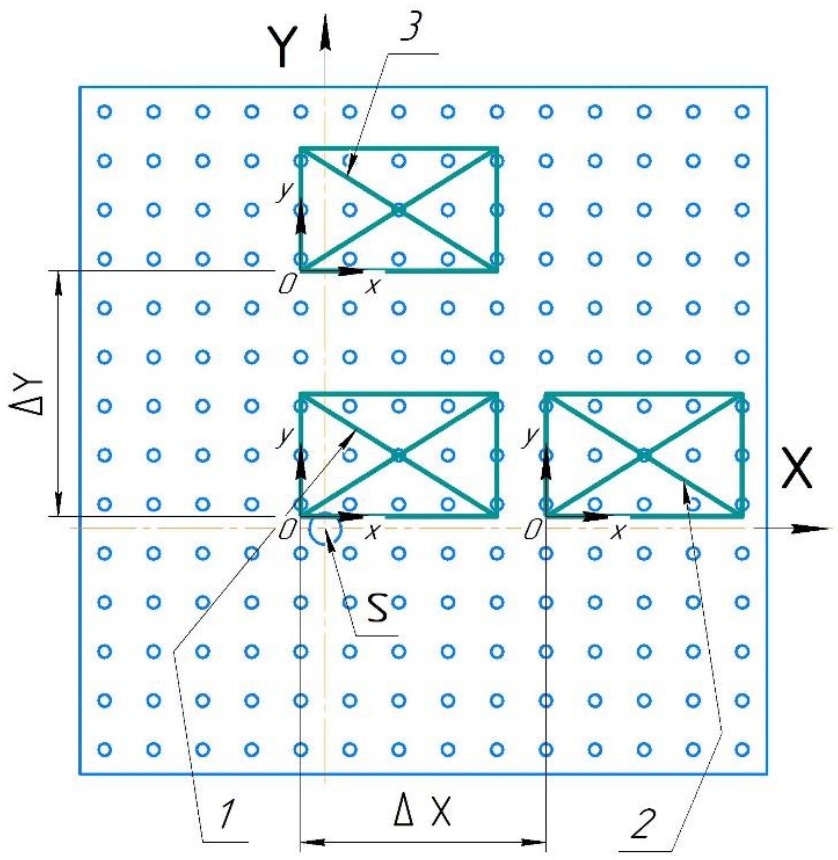

Figure 1. The scheme of the rotating plate of the torsional platform

Thus, we obtained formulas for estimation of three parameters, namely, I_z, x_C, y_C.
In order to obtain I_x, I_y,z_C we have to put the nanosatellite in the corresponding position on the torsional platform.

For estimation products of inertia, we use moment of inertia method. We suggest to measure the moments of inertia along three axes which form the 45° with the positive direction of *x, y, z* axes. And then calculate products of inertia using formulas

$$I_{xy} = \frac{(I_x + I_y)}{2} - I_4, \quad I_{xz} = \frac{(I_x + I_z)}{2} - I_5, \quad I_{yz} = \frac{(I_y + I_z)}{2} - I_6. \quad (14)$$

Here I_4 is measured along axis between positive direction *x* and *y* axes; I_5 is measured along axis between positive direction *x* and *z* axes; I_6 is measured along axis between positive direction *y* and *z* axes.

Description of an instrumented torsional platform

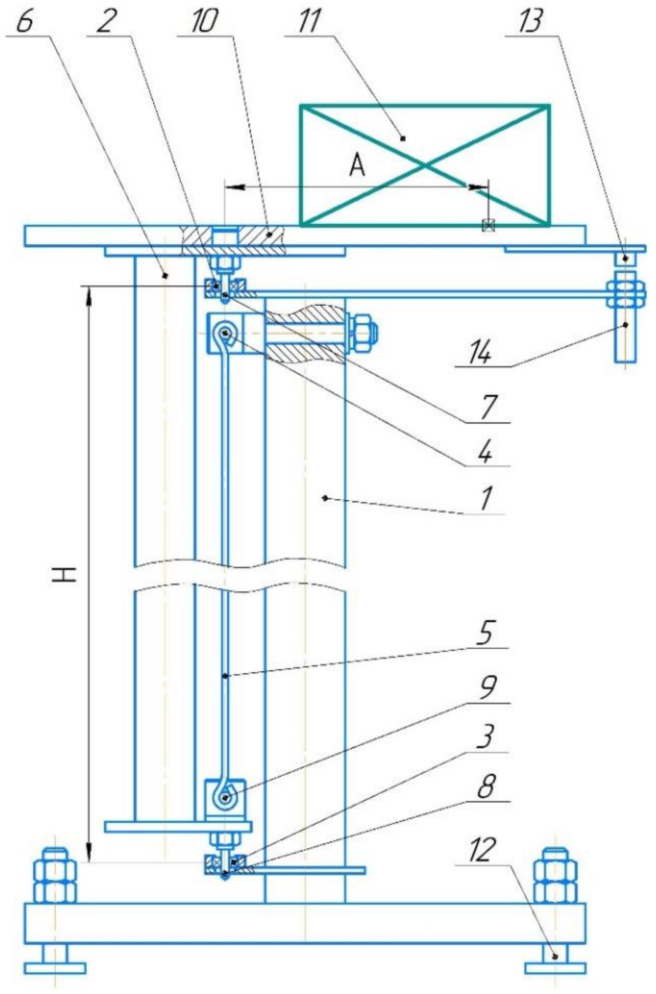

Figure 2. The scheme of the torsional platform

The proposed torsional platform consists of a frame 1 and a balance beam (a rotation shaft with a rotation plate). On the frame 1, the upper 2 and lower 3 radial rolling bearings are installed, located coaxially. Under the bearing 2 attachment unit 4 is installed to attach the torsion spring 5. The upper 7 and lower 8 spikes are installed on the shaft 6. The attachment point 9 of the torsion spring 5 is at the upper part of the spike 8. The upper part of the shaft 6 has a rotating plate 10.

The rotating plate has a pattern of coordinate holes for installing the object 11. The frame 1 has adjustable screw supports 12. There is the arrow 13 on the torsional platform plate 10, and the optical encoder 14 is installed on the frame.

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Figure 3. The torsional platform

Prevention of accidents

1. Only the persons who have studied the operating manual, who have undergone introductory briefing, initial instruction on labor protection at the workplace, and have no medical contraindications are allowed to work with the stand.
2. It is forbidden to disassemble the stand or carry out repair works without disconnecting it from the power supply.
3. It is forbidden to use modes not provided by the operating manual.
4. It is forbidden to connect the timer to the power supply in the presence of visible damage to the socket, plug or connecting cord.
5. It is forbidden to carry the stand in the assembled state! If it is necessary to carry the stand, in order to avoid damage to the bearings, it is necessary to remove the lower mount of the balancer and remove the balancer from the frame!
6. It is forbidden to use the stand if the timer power cord is damaged.
7. It is forbidden to measure the characteristics of objects whose mass exceeds 10 kg.
8. It is forbidden to tilt the work table of the stand by more than 40˚.
9. It is forbidden to use the surface of the torsional platform for storing any objects.
10. The attachment blocks, the object under study and the standards located on the working table must be securely pinned against displacement, and in the case of their hanging from the working table, secured against overturning.
11. Remove (insert) the memory card only when the timer is switched off from the network.

Procedure of experimental evaluation of nanoclass spacecraft design parameters:

- Adjustment and calibration;
- Estimation of the center of mass coordinates;
- Estimation of the inertia tensor;
- Estimation of errors.

Adjustment and calibration

Preparation of the platform for operation consists in setting the verticality of the axis of rotation and determining the spring constant for various loads on the plate and tipping moments.

Calibration is determination the constant spring for various masses using the following procedure:

1. Set the standard of mass m (taking into account the mass of the attachment finger) to the nearest diagonal coordinate hole to the axis of rotation (distance r_1). Measure the period of oscillation T_1
2. Shift the standard to the next hole from the axis of rotation along diagonal (the distance r_i). Measure the period of oscillation T_i. Repeat step 2 for the next holes
3. Calculate the value of spring constants by formulas

$$k_S = \frac{4\pi^2 m\left(r_i^2 - r_j^2\right)}{T_i^2 - T_j^2}, \quad i = 2, \dots, n, \quad j = 1, \dots, (n-1) \qquad (15)$$

4. Calculate the average value of spring constant.

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Estimation of the moments of inertia

The moments of inertia are determined with the known center of mass coordinates of the nanosatellite (x_C, y_C, z_C).

When determining the moments of inertia, the attachment blocks must be installed on the small rotation plate in such way that the center of mass of the nanosatellite is as close as possible to the axis of rotation.

1. Install on the small rotation plate the vertical attachment blocks. Measure the oscillation period of the torsional platform with attachment blocks T_1.
2. Install the nanosatellite in the attachment blocks so that the *x*-axis is directed vertically, the direction of the z-axis coincides with the direction of the Y-axis of the working table. Using the drawings of the attachment blocks determine the projection of the center of mass of the nanosatellite X_{C1}, Y_{C1} on the plane of the torsional platform. Measure the period T_2.
3. Calculate the moment of inertia of the nanosatellite I_x by the formula

$$I_x = \frac{\left(T_2^2 - {T_1}^2\right) \cdot k_S}{4\pi^2} - m(X_{C1}^2 + Y_{C1}^2). \qquad (16)$$

4. Install on the small rotation plate the diagonal-horizontal attachment blocks. Measure the oscillation period of the torsional platform with attachment blocks T_3.
5. Install the nanosatellite in the attachment blocks, as shown in Figure 4, so that the directions of the x and y axes of the nanosatellite's building coordinate system coincide with the directions of the same X and Y axes of the torsional platform. Using the coordinates of the center of mass of the object, using a vernier depth gauge and a known distance (75.7 mm) from the axis of rotation of the to the base plane, set the nanosatellite in a such way that the projection of its center of mass onto the torsional platform is as close as possible to the corresponding projection of the axis of rotation. Determine the projection of the center of mass of the nanosatellite X_{C2}, Y_{C2} on the plane of the torsional platform Measure the oscillation period of the torsional platform with attachment blocks T_4.

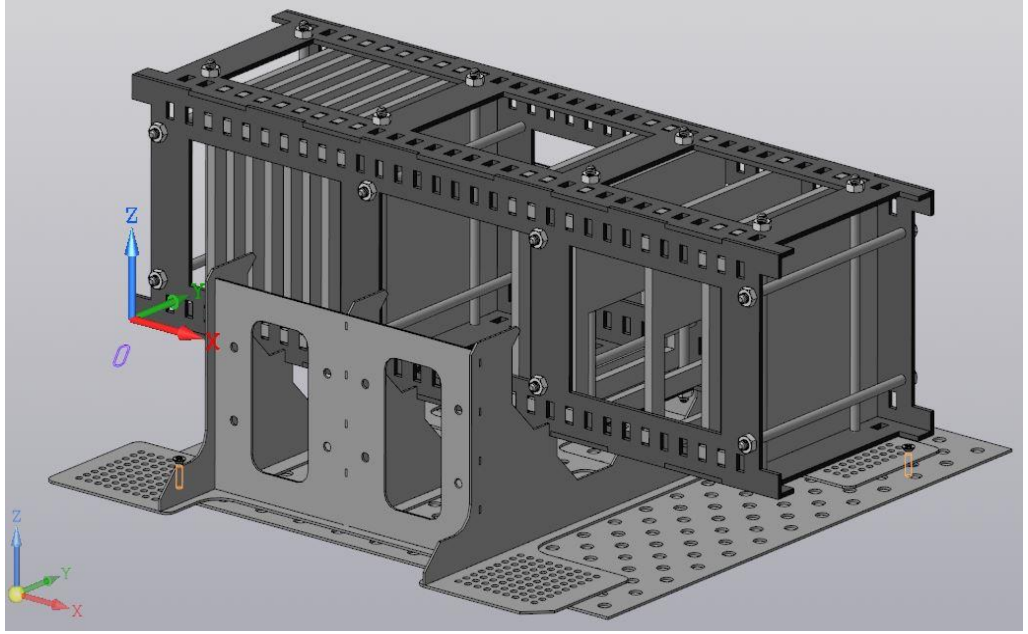

Figure 4. The initial direct position of the nanosatellite in a diagonal-horizontal attachment block for calculating the axial moment of inertia I_z

6. Install the nanosatellite in the attachment blocks leaving unchanged the displacement of the nanosatellite center of mass along the X-axis of the torsional platform. It is necessary to rotate the nanosatellite by 90 ° around its longitudinal x-axis so that the positive direction of the z-axis of the building coordinate system associated with the nanosatellite coincides with the positive direction of the Y-axis of the torsional platform. Determine the projection of the center of mass of the nanosatellite X_{C3}, Y_{C3} on the plane of the torsional platform. Measure the oscillation period of the torsional platform with attachment blocks T_5.
7. Calculate the moment of inertia of the nanosatellite I_z by the formula

$$I_z = \frac{\left(T_4^2 - {T_3}^2\right) \cdot k_S}{4\pi^2} - m(X_{C2}^2 + Y_{C2}^2). \qquad (17)$$

8. Calculate the moment of inertia of the nanosatellite I_y by the formula

$$I_y = \frac{\left(T_5^2 - {T_3}^2\right) \cdot k_S}{4\pi^2} - m(X_{C3}^2 + Y_{C3}^2). \qquad (18)$$

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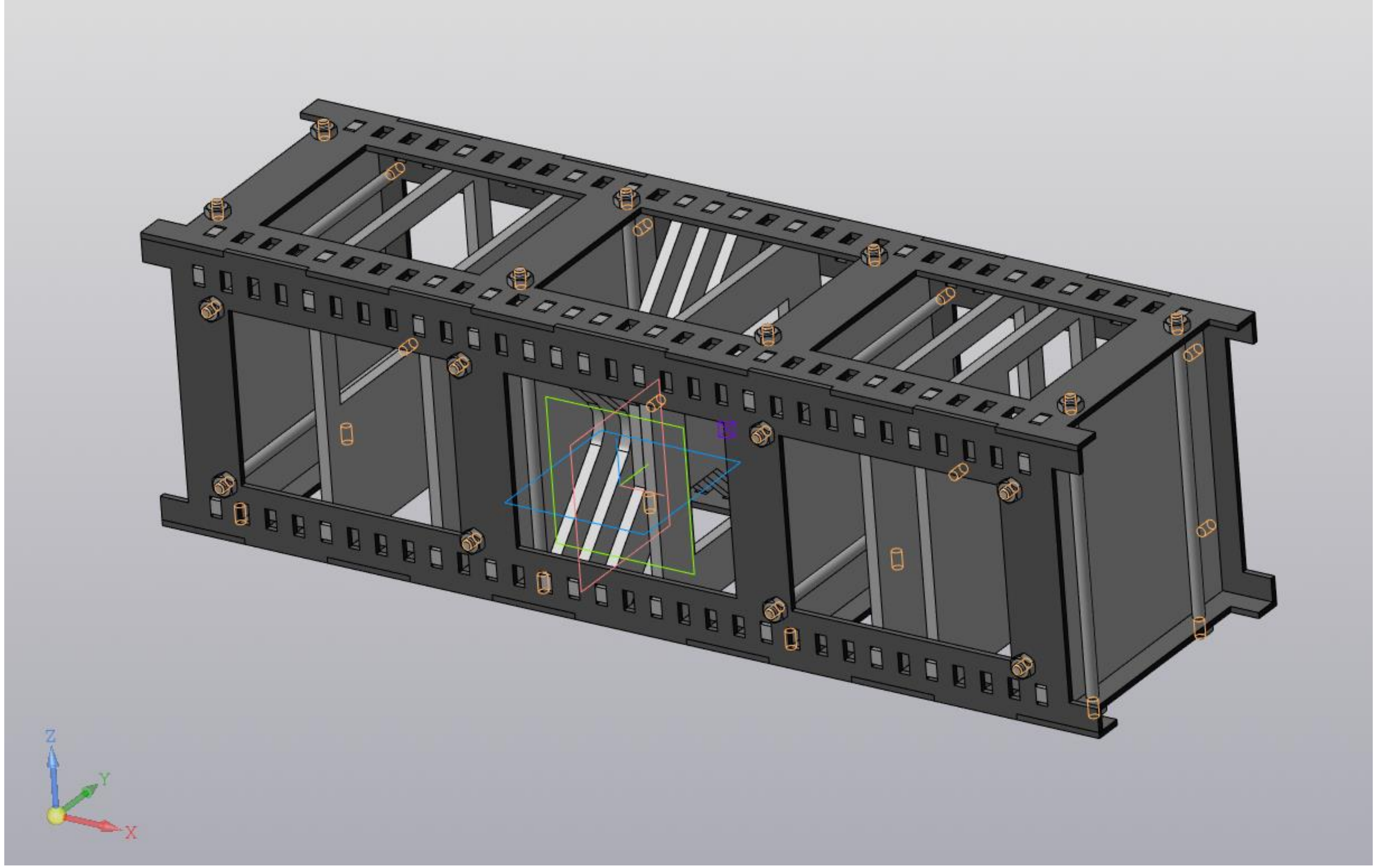

CAD values

Mass:

$$m = 3.5814 \text{ kg.}$$

The center of mass coordinates in building coordinate system

$$x_c = 167.82 \text{ mm};$$
$$y_c = 48.95 \text{ mm};$$
$$z_c = 49.48 \text{ mm}.$$

Moments of inertia in Centroid coordinate system:

Axial moments of inertia:

$$I_x = 0.007819 \text{ kg·m}^2,$$
$$I_y = 0.037088 \text{ kg·m}^2,$$
$$I_z = 0.037085 \text{ kg·m}^2.$$

Products of inertia:

$$I_{xy} = -0.000386 \text{ kg·m}^2;$$
$$I_{xz} = 0.000202 \text{ kg·m}^2;$$
$$I_{yz} = -0.000002 \text{ kg·m}^2.$$

Formulas for calculation

$$I_x = \frac{\left(T_2^2 - T_1{}^2\right) \cdot k_S}{4\pi^2} - m(X_{C1}^2 + Y_{C1}^2).$$

$$I_z = \frac{\left(T_4^2 - T_3{}^2\right) \cdot k_S}{4\pi^2} - m(X_{C2}^2 + Y_{C2}^2).$$

$$I_y = \frac{\left(T_5^2 - T_3{}^2\right) \cdot k_S}{4\pi^2} - m(X_{C3}^2 + Y_{C3}^2).$$

If the axes x, y of the Reference coordinate system and the axis X, Y of the torsional platform coordinate system are in the same direction, we can estimate the values x_C, y_C by

$$X_C = X_O + x_C,$$

$$Y_C = Y_O + y_C,$$

where X_O, Y_O are origin *O* coordinates in the torsional platform coordinate system.

Spring constant k_S=0.57083 N·m

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Formulas for calculation

$$I_x = \frac{\left(T_2^2 - {T_1}^2\right) \cdot k_S}{4\pi^2} - m(X_{C1}^2 + Y_{C1}^2).$$
$$I_z = \frac{\left(T_4^2 - {T_3}^2\right) \cdot k_S}{4\pi^2} - m(X_{C2}^2 + Y_{C2}^2).$$
$$I_y = \frac{\left(T_5^2 - {T_3}^2\right) \cdot k_S}{4\pi^2} - m(X_{C3}^2 + Y_{C3}^2).$$

If the axes x, y of the Reference coordinate system and the axis X, Y of the torsional platform coordinate system are in the same direction, we can estimate the values x_C, y_C by

$$X_C = X_O + x_C,$$
$$Y_C = Y_O + y_C,$$

where X_O, Y_O are origin O coordinates in the torsional platform coordinate system.

$$T_1 = 1311.70\ ms, T_2 = 1503.80\ ms,$$
$$T_3 = 1352.64\ ms, T_4 = 2095.57\ ms,$$
$$T_5 = 2090.88\ ms,$$

Spring constant k_S=0.57083 N·m

Mass:

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